String Theory and Strong Interactions

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Texas A&M, April 19, 2004

Fundamental questions of high energy physics

Understanding of the Universe relies on knowing:

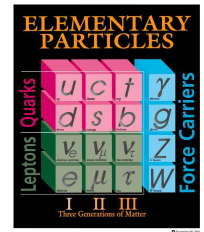
- the nature of the building blocks
- the nature of the forces through which they interact

The last $\sim$ 30 years:

- Standard Model of particle physics: well tested by experiments
- Gravity

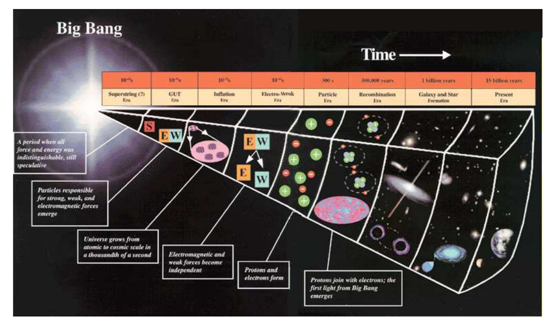
String theory

- understand physics at high energy
- consistent theory of quantum gravity
- aims to explain in unified way all particles and interactions



- black holes: explain entropy, radiation, information paradox, etc

- cosmology: singularities, cosmological constant, etc.



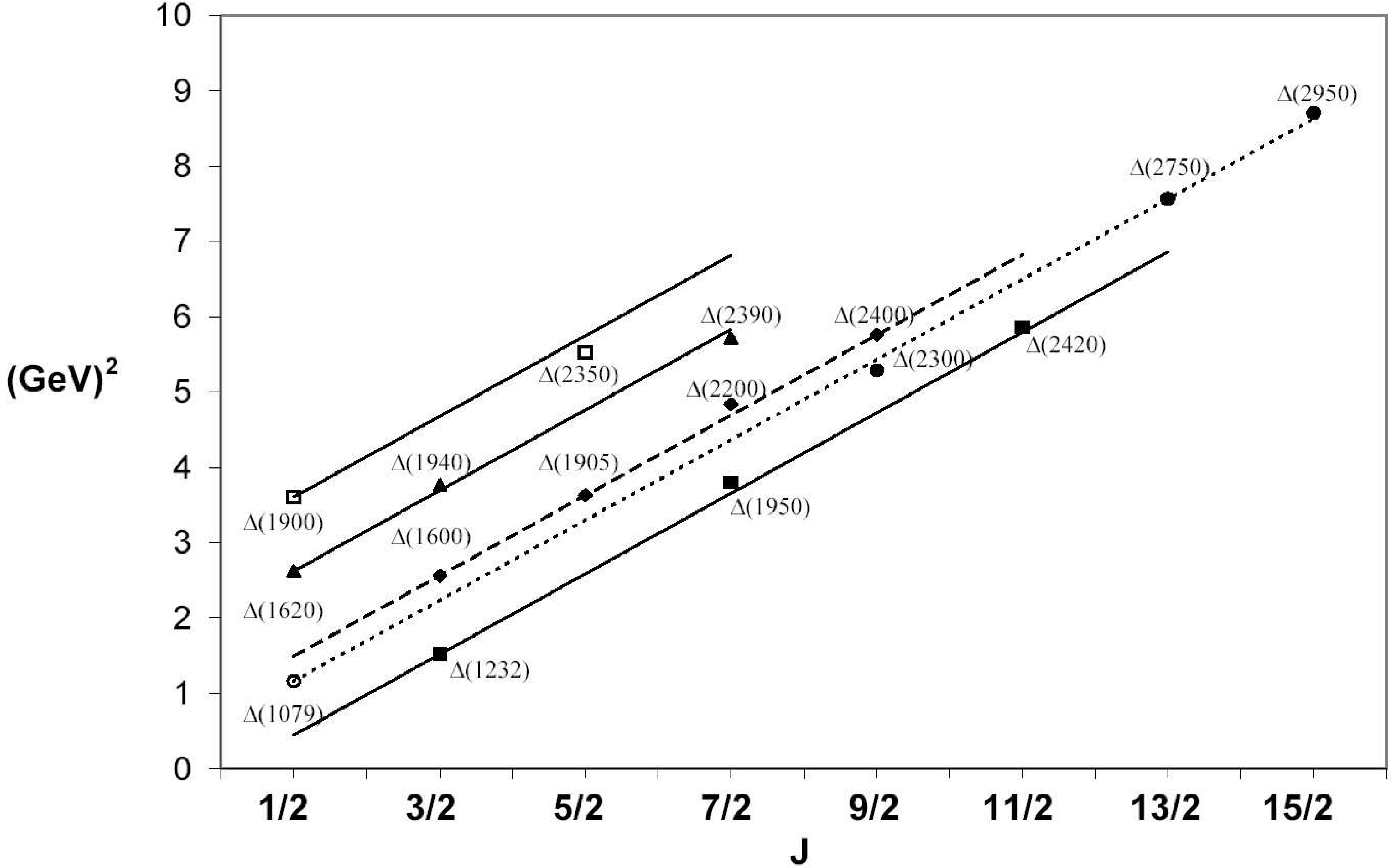
- learn about strong interactions (gauge theory/QCD)

History

Strong interactions: 1960's

No theory

- Various models: each described some aspects of hadronic physics:
  - Short range $\longrightarrow$ no massless particles
  - Symmetries and conservation laws (C, P, T, I, ...)
  - Ever-growing number of metastable states – resonances extending to large spin — a zoo?
- quantum field theory (QFT) seemed hopeless
  - too many (increasing with time) degrees of freedom
  - high spin QFT's are usually non-renormalizable



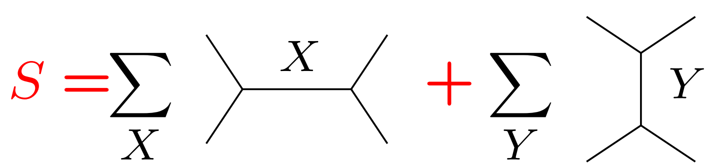
(GeV)^2
10
9
8
7
6
5
4
3
2
1
0
1/2
3/2
5/2
7/2
9/2
11/2
13/2
15/2
J
Δ(2950)
Δ(2750)
Δ(2390)
Δ(2400)
Δ(2420)
Δ(2300)
Δ(2350)
Δ(2200)
Δ(1940)
Δ(1905)
Δ(1900)
Δ(1950)
Δ(1600)
Δ(1620)
Δ(1232)
Δ(1079)

Strong interactions $\longrightarrow$ string theory

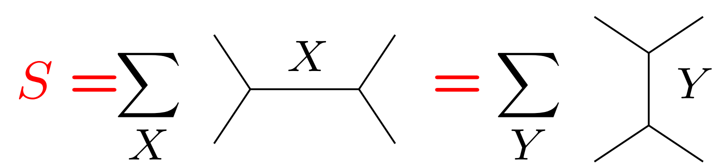
- Analytic S-matrix
  - Only on-shell scattering amplitudes are physical
  - Unitarity, analyticity, Lorentz invariance, etc
  - bootstrap conjecture (exchange of hadrons mediates strong interactions)
  - analyticity in angular momentum

Consequence: "duality"

field theory



hadrons



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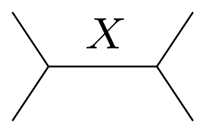
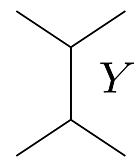
Consequence: "duality" $\sum_X$ 

$= \sum_Y$ 

- A solution: Veneziano amplitude ($2 \to 2$ scattering)

$$A = A(s,t) + A(s,u) + A(t,u) \qquad A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))}$$

$$\alpha(s) = a + \alpha' s$$

$\rightarrow$ scattering amplitudes of modes of a relativistic string

Is it a bug or a feature?

- quantization requires more than 4 dimensions
- against observation of pointlike substructure of hadrons
- spectrum contains massless spin 1 and spin 2 states
- hard to go off-shell

"Low energy" limit: Einstein-Hilbert action (GR) + more

Became a unified theory of quantum gravity and other interactions

String theory

- understand physics at very high energy
- unification of all interactions (gauge + gravity)
- black holes: explain entropy, radiation, information paradox, etc.
- cosmology: singularities, cosmological constant, etc
- ...
- learn about strong interactions (gauge theory/QCD)

In the meantime – new theory of strong interactions: QCD

- renormalizable QFT
- ultraviolet freedom – reliable computations at high energies
- conjectured IR confinement

Different from the theories studied before:

- fundamental fields are different from the observed particles
- can account for the large number of resonances

Part of the Standard Model: $SU(3) \times SU(2) \times U(1)$

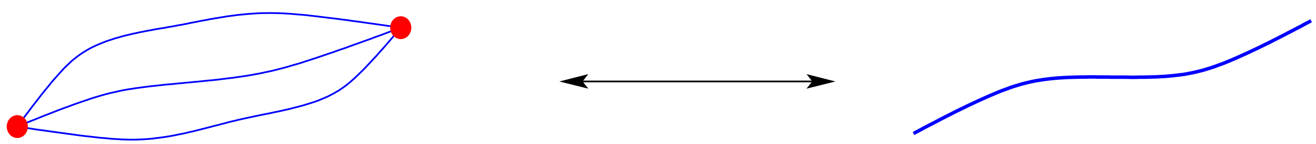
- quarks and gluons

Still open problems, both conceptual and technical:

- Is there a string theory of strong interactions?
  - insight into nonperturbative physics
- Understand confinement
  - quarks and gluons $\longrightarrow$ zoo of hadrons
- Efficiently compute processes relevant to collider physics (LHC)
  - distinguish new physics

The QCD string?

- Yang Mills theory; $SU(N_c)$

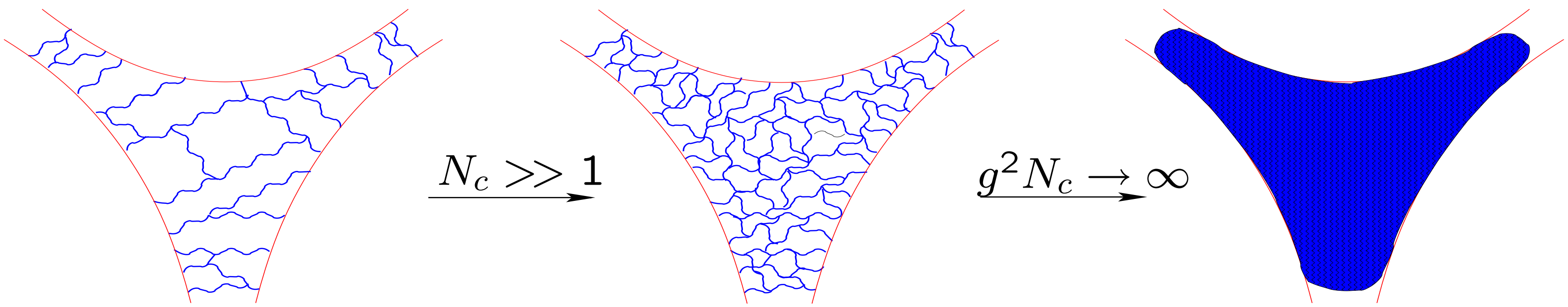


QCD meson — Open string

- flux tubes of color field act as strings
- intuitive explanation of confinement

- 't Hooft 1974:

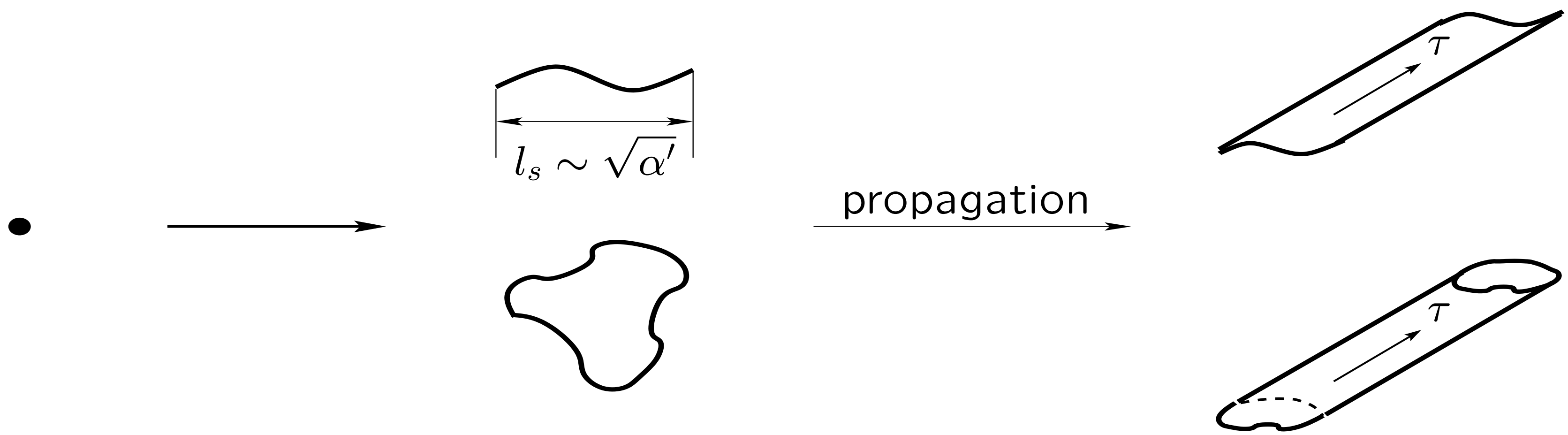
- taking the large N_c limit keeping $\lambda = g^2_{YM} N_c$ fixed gives a smooth limit: only planar graphs survive; no quark loops



- $\lambda >> 1$ the gluons form a "continuous" surface

$\Rightarrow$ Is it a string theory?

String Theory



X^μ	scalar fields		coordinates
	world sheet	vs.	space-time
	2d QFT	$S = \int d^2z\, g_{\mu\nu} \partial X^\mu \bar{\partial} X^\nu$	$S \sim$ area
$g_{\mu\nu}$	coupling constants		metric
	internal symmetries		symmetries

String Theory

- quantization: for the supersymmetric (SUSY) string d=10

  SUSY: bosons $\longleftrightarrow$ fermions

  parameters: string length $\sqrt{\alpha'}$

Controls the spectrum

- Solve eom for the world sheet metric ($E_{ws} = a$)

e.g. flat space

$$L = \frac{1}{\alpha'}(\partial X)^2 \quad \rightarrow \quad H = \frac{1}{\alpha'}\left((\partial_0 X)^2 + (\partial_1 X)^2\right)$$

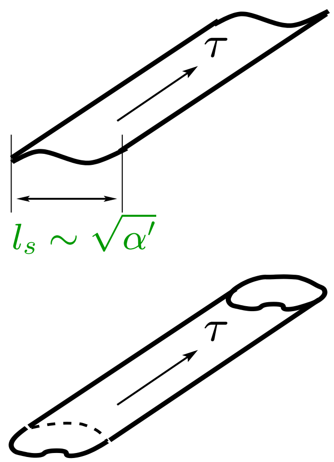
$$E_{ws} = \alpha' p^2 + \sum_{n \geq 1} a_n n \quad \longrightarrow \quad -p^2 = m^2 = \frac{1}{\alpha'}\left(-a + \sum_{n \geq 1} a_n n\right)$$

String Theory

- quantization: for the supersymmetric (SUSY) string d=10

  SUSY: bosons $\longleftrightarrow$ fermions

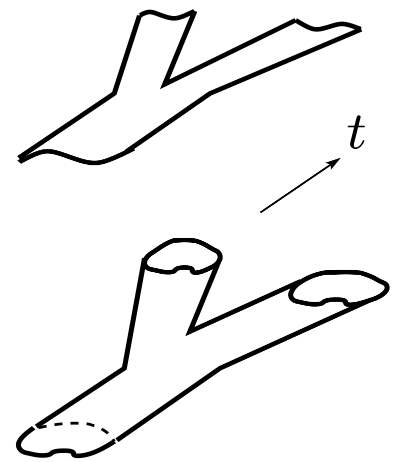
  parameters: string length $\sqrt{\alpha'}$ and coupling constant g_s



Spectrum: gauge fields (YM)
+ more

interactions; g_s $\longrightarrow$

Spectrum: metric (GR)
+more



- there are 5 consistent theories: IIA, IIB, Type I, 2 $\times$ Heterotic

  all related by dualities

Dualities

- Familiar example:

– Electromagnetic duality (no charged particles)

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = -\frac{\partial \vec{E}}{\partial t}$$

$\vec{E} \to \vec{B}$, $\vec{B} \to -\vec{E} \Rightarrow$ same equations, equivalent physics

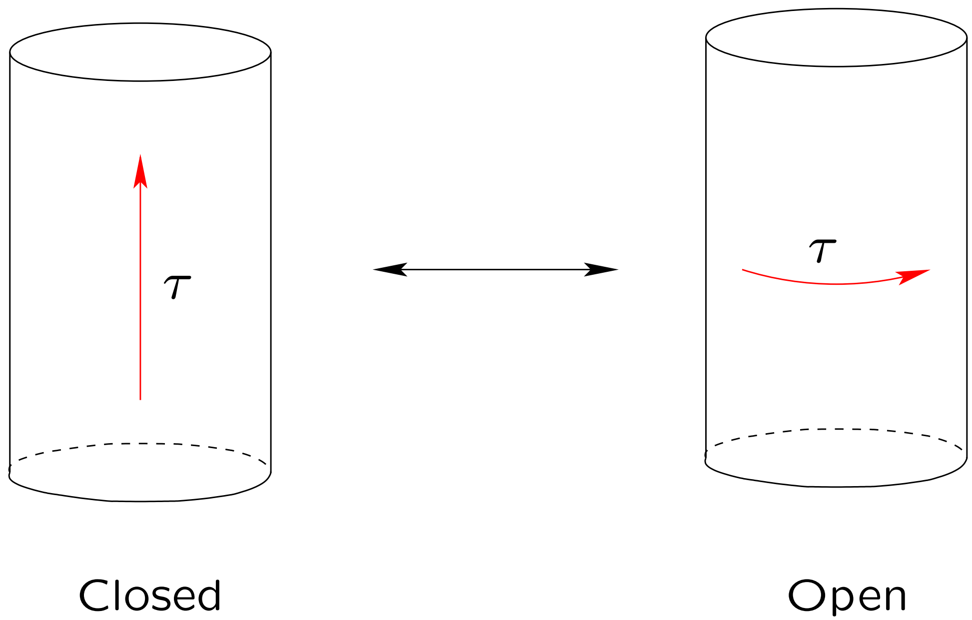
Definition: $H = H_0 + gH_{int} = H'_0 + g'H'_{int}$ $\qquad g' = \frac{1}{g}$

- Field theory:
  - Sine-Gordon vs. Thirring model in 2 dimensions
  - S-duality/electric-magnetic duality (Montonen, Olive)
    - at the basis of the Seiberg-Witten solution of $N = 2$ theories
  - Dualities in supersymmetric field theories relating the field content and the rank of the gauge group

(Seiberg, Intriligator, Kutasov, etc)

- String theory:

– open-closed/world sheet duality (the dawn of string theory)



- String theory:

– open-closed/world sheet duality (the dawn of string theory)

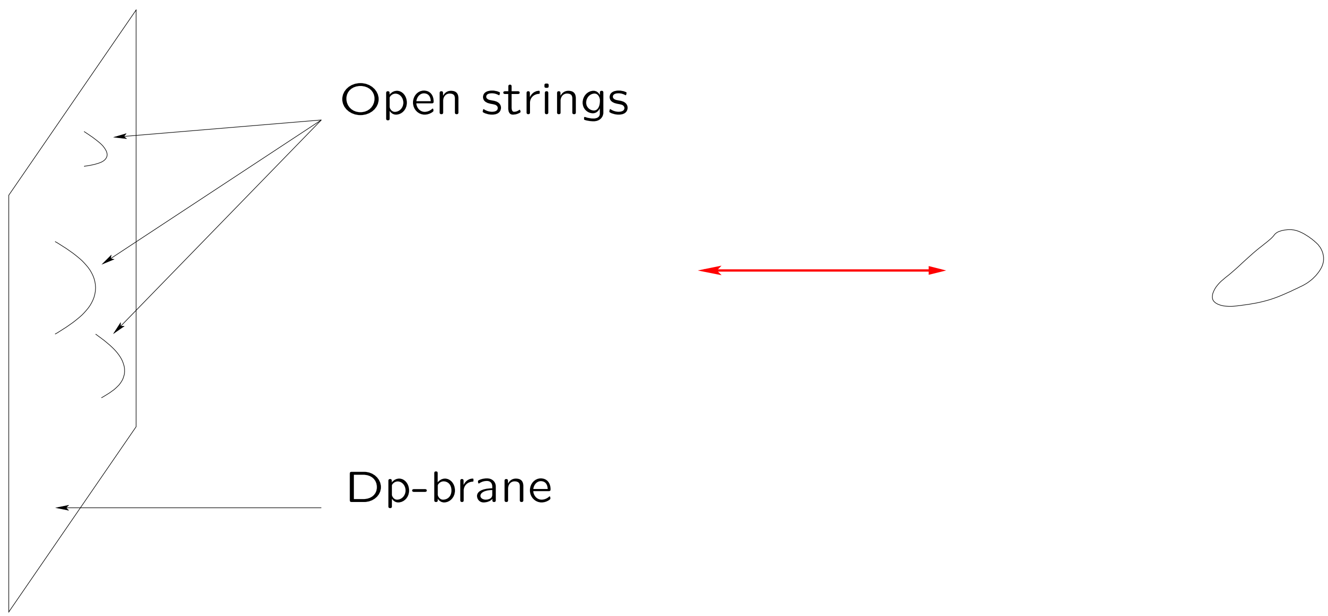
– $SL(2,\mathbb{Z})$ – analog of S-duality

– T-duality (Bucher)

– Brane constructions $\longrightarrow$

– Many other dualities discovered/conjectured since 1994

  – among them: black hole – D-brane duality


Open strings
Dp-brane

Some recent progress

Plan

- **AdS/CFT**:
  - hidden symmetries for strings in $AdS_5 \times S^5$
    $\longrightarrow$ solution to a large N gauge theory
  - **Spinoff**: spin chains and anomalous dimensions in field theories
- **QCD amplitudes from topological strings**

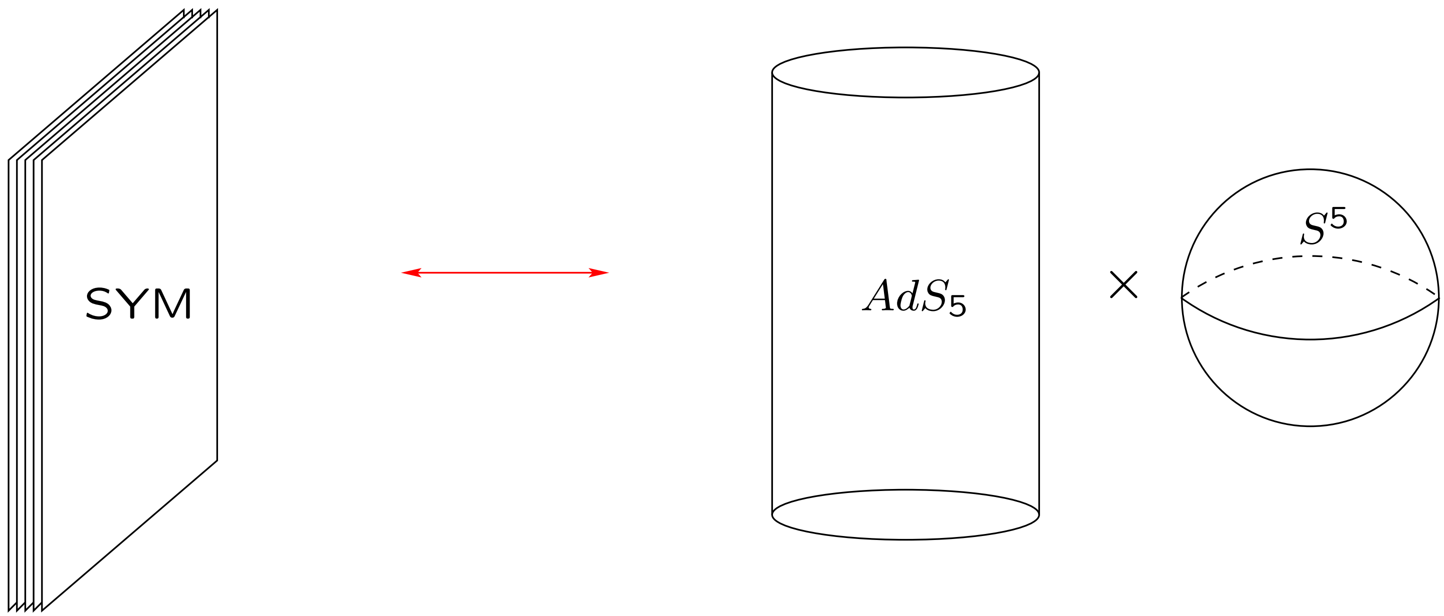
Anti de Sitter/Conformal Field Theory correspondence

AdS/CFT

Maldacena 1997: AdS/CFT correspondence

- low energy limit of the black hole – D-brane duality

$$\mathcal{N} = 4 \text{ U(N) super YM} = \text{IIB string in } AdS_5 \times S^5$$



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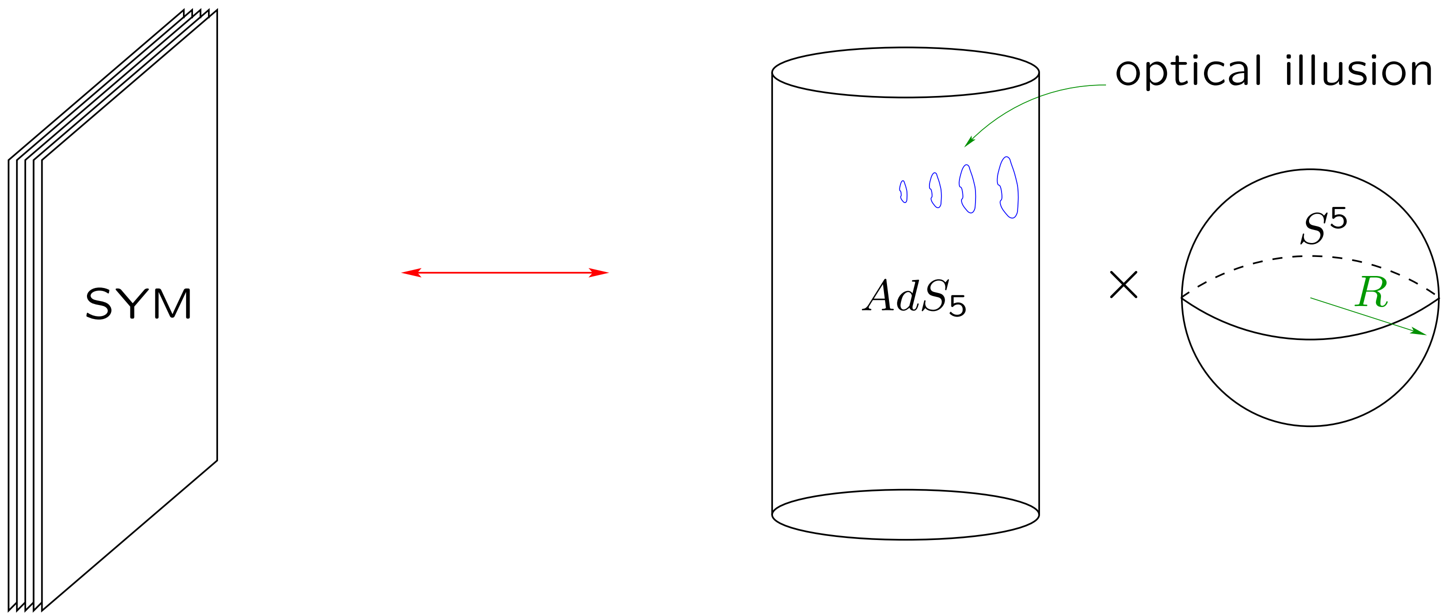

$$\begin{pmatrix}\text{gluons, 6 scalars}\\ \text{4 Weyl fermions}\end{pmatrix}_{adj} \qquad g^2_{_{YM}} = g_s \qquad \begin{pmatrix} ds^2 = \frac{r^2}{R^2}dx^2_{(4)} + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2 \\ R^4 = (g^2_{_{YM}}N)l_s^4 = \lambda l_s^4 \end{pmatrix}$$

$$\mathcal{L}_{SYM} = Tr[F^2] + \begin{pmatrix}\text{scalars}\\ \text{fermions}\end{pmatrix} \qquad \mathcal{L}_{SUGRA} = \text{R} + (\text{other fields})$$

Maldacena 1997: AdS/CFT correspondence

$\mathcal{N}=4$ $U(N)$ super YM	$=$	IIB string in $AdS_5 \times S^5$
quantum field theory	$\longleftrightarrow$	gravitational theory
$[g_{YM} \to 0 \quad N \to \infty \quad \lambda \to \infty]$ open string	$\longleftrightarrow$	$\left[R_{AdS} \propto \lambda^{1/4} \to \infty \quad g_s \to 0\right]$ closed string/SUGRA
quantum	$\longleftrightarrow$	classical (world sheet & space – time)
gauge invariant operators traces of products of fields	$\longleftrightarrow$	string states
global symetries ($4d$ superconformal group)	$\longleftrightarrow$	isometries of $AdS_5 \times S^5$

The slogan: AdS/CFT partly realizes 't Hooft's idea by describing a strongly coupled YM theory as a closed string theory

- countless tests in the strict $N \to \infty$ limit

- symmetries, moduli space, correlation functions of BPS operators, anomalies

- use SUGRA: $g_s \to 0$, $R^4_{AdS} \propto \lambda \to \infty$ YM coupling large at all scales

- Physics is very different from QCD

- no asymptotic freedom, confinement/chiral symmetry breaking

- Generalizations which have such qualities

- Tests of the correspondence at finite λ: plane wave limits
  – works incredibly well

Idea:
- At finite AdS radius
- Expand the world sheet theory around the trajectory of a massless particle in $AdS_5 \times S^5$
- Take AdS radius to infinity with R^2/J fixed
- Can keep g_s finite

- Describes a special class of operators in $\mathcal{N} = 4$ SYM

(Berenstein, Maldacena, Nastase)

Recovered all string theory predictions for the properties of these operators (Gross, Mikhailov, Roiban)

But....

... can one use AdS/CFT to solve $\mathcal{N} = 4$ SYM?

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$g_{YM} \to 0$ $\lambda-$arbitrary $\longrightarrow$ $g_s \to 0$ $R^4_{AdS} \propto \lambda-$arbitrary

Solve the world sheet for any R^4_{AdS} $\longleftrightarrow$ Solve planar $\mathcal{N}=4$ SYM

Simplest output: spectrum   More complicated: correlation functions

Consequence: Proof of AdS/CFT

- describe planar, weakly coupled YM as a closed string theory

Observation:

$R_{AdS} \to 0$: S^5 shrinks to 0 size $\to$ string lives in a space of its size – 100% quantum

The solution to this problem may lead to interesting physics/math with other applications, e.g. string theory near cosmological singularities

Replace strongly coupled 4d theory with strongly coupled 2d theory

(ws theory for superstrings in $AdS_5 \times S^5$) **Is this progress?**

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Best current answer: Likely to be progress

There exist nonperturbative techniques valid only in two dimensions

- More familiar: current algebras/holomorphic currents
  - Sugawara construction of the stress tensor
  - spectrum related to quadratic Casimir

  Unfortunately not applicable due to details of the theory

- Less familiar: Bethe ansatz/integrable models
  - have reason to believe that theory is solvable
  - hallmark: infinitely many integrals of motion

The superstring on $AdS_5 \times S^5$ has an infinite number of conservation laws which form a Yangian algebra (Bena, Polchinski, Roiban)

Examples:

- symmetries of $AdS_5 \times S^5$: $J^A = \int dx\, \mathcal{J}_\tau^A(x)$
- nonlocal: $Q_2 = \int dx \int_x dy\, \mathcal{J}_\tau(x)\mathcal{J}_\tau(y) + \text{(local)}$

The good news:

- Simmilar structures appear in standard integrable QFT-s
  (Lüscher, Pohlmeyer, Curtright, Zachos, de Vega, Eichenherr, etc)
- Extend to the quantum level
- Factorizable S-matrix $\longrightarrow$ Bethe ansatz $\longrightarrow$ (part of) spectrum
  (Lüscher) (Polyakov, Wiegmann, Reshetikhin, Faddeev, etc)

“A solvable model is not solvable until it is solved” (Yogi Berra)

Each integrable model has its own trick

Example: sigma model on (flat space) $\times S^5$

$$\mathcal{L} = \sum_i \partial X^i \bar{\partial} X_i + \sum_{j=1}^{6} \partial Y^j \bar{\partial} Y_j$$

Example: sigma model on (flat space) $\times S^5$

$$\mathcal{L} = \sum_i \partial X^i \bar{\partial} X_i + \sum_{j=1}^{6} \partial Y^j \bar{\partial} Y_j + \mu(\sum_j Y^j Y_j - 1)$$

$$H = H_{flat}(X) + H_{S^5}(Y) \;\rightarrow\; E = \left(p^2 + \sum_{n_i} a_{n_i} n_i\right) + E_{S^5}$$

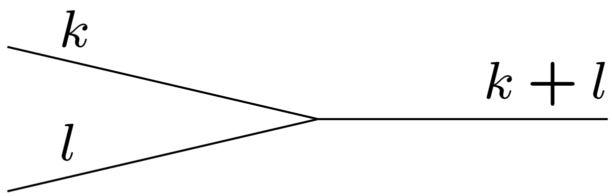
Idea: reduce everything to free particles

labeled by S^5 analog of spin quantum numbers

- the model posesses J^a and Q_2^a – limits of previous ones
- $2d$ translational invariance: $P = m_k e^{\theta}$ and $\bar{P} = m_k e^{-\theta}$

• analyze the conservation of P, $\bar{P}$, $\sum_a J^a J^a$ and $\sum_a J^a Q_2^a$

- $(J_k^a)^2|k\rangle = k(k+4)|k\rangle$



momentum conservation

$$\downarrow$$

$$m_k \sinh\theta_k + m_l \sinh\theta_l = 0$$

$$\left.\begin{array}{l}\text{conservation of } \sum_a J^a J^a \\ \text{conservation of } \sum_a J^a Q_2^a\end{array}\right\} \longrightarrow \quad k\theta_k + l\theta_l = 0$$

$$\downarrow$$

$$\frac{m_k}{m_l} = \frac{\sin\alpha k}{\sin\alpha l} \quad \Rightarrow \quad m_k = M \sin\frac{k}{4}$$

$$E = \left(p^2 + \sum_{n_i} a_{n_i} n_i\right) + M^2 \sin^2\tfrac{k}{4}$$

Relation to gauge theory

AdS/CFT: Q_2 (and higher) should have a gauge theory analog

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AdS/CFT: Q_2 (and higher) should have a gauge theory analog

Test: SYM exhibits a different kind of integrable structure: 1-loop scale transformations of gauge invariant operators can be described in terms of an integrable spin chain (Minahan, Zarembo, Beisert, Staudacher)

– Construct the Yangian algebra (Dolan, Nappi, Witten; Belavin; Drinfeld)
The two integrable structures are compatible

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Spinoff: Can formulate general relation between integrable spin chains and field theories, in the absence of a string description?

Yes: For any integrable spin chain there exists a field theory whose 1-loop dilatation operator acts on gauge invariant operators as the Hamiltonian of the spin chain. (Roiban)

Many open problems

- use integrals of motion to classically solve the ws theory
- classify gauge theory Wilson loops
- use Bethe ansatz type techniques to solve the quantum theory
- extend to more realistic theories
- ...

QCD and topological strings

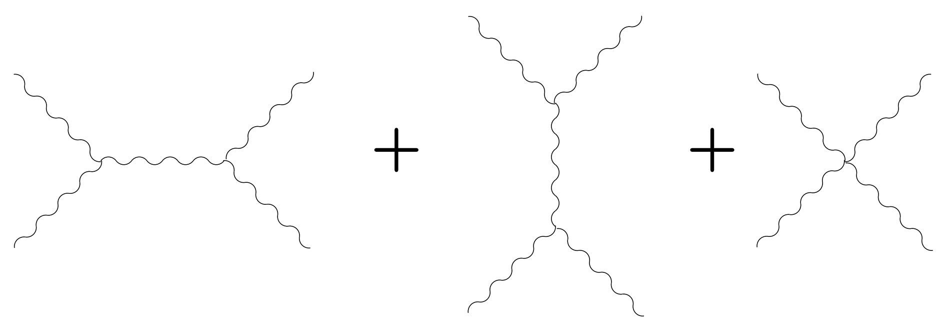
- AdS/CFT: string excitations $\longleftrightarrow$ gauge invariant operators
- Open string theory: SYM + infinitely many massive fields

Calculation of QCD amplitudes $\longrightarrow$ collider phenomenology

Question: Is there a string theory whose spectrum and interactions are those of a Yang Mills theory?

Witten: Yes. for $\mathcal{N} = 4$ SYM

Tree level scattering amplitudes are the same as in QCD, e.g. $gg \to gg$



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Conjectured string theory :
$$\begin{cases} \text{for } \mathcal{N} = 4 \text{ SYM}: \ \text{lives on } CP^{3|4} \\ \text{open} \\ \text{finite number of fields} \leftrightarrow \text{topological} \\ \text{complete}: \ \text{perturbative} + \text{nonperturbative} \end{cases}$$

Tests:

Recover examples of tree level YM amplitudes

(Roiban, Spradlin, Volovich)

Recover propertires of tree level YM amplitudes

(Roiban, Spradlin, Volovich;Witten; Berkovits, Motl; Gukov, Nietzke)

	string theory	gauge theory
$A(1,2,\ldots,n) = A(2,3,\ldots,1)$	manifest	non-manifest
$A(1,2,\ldots,n) = (-)^n A(n,n-1,\ldots,1)$	manifest	non-manifest
CPT	non-manifest	manifest
$\sum_{C(1,\ldots,n-1)} A(1,2,\ldots,n) = 0$	manifest	non-manifest
$\sum_{perm(i,j)} A(i_1,\ldots,i_m,j_1\ldots,j_k,n) = 0$	manifest	non-manifest
factorization	non-manifest	manifest

Tests and consequences:

Recover examples of tree level YM amplitudes (RR, Spradlin, Volovich)

Recover propertires of tree level YM amplitudes (RR, Spradlin, Volovich)
(Witten; Berkovits, Motl)

New (different) prescription for computing tree level YM amplitudes
(Cachazo,Svrcek, Witten)

Hope:

These new techniques pass all further tests and become computationally useful for phenomenologically interesting processes

Summary

String Theory is useful!

- gauge theory
  - solution of large N_c SYM
  - QCD amplitudes
  - nonperturbative info about SUSY Yang-Mills
- understand physics at very high energies
- unification of all interactions
- brane worlds/extra dimensions
- black hole physics
- cosmology
- ...

The idea of the construction:

Flat current: $A = A_\tau d\tau + A_\sigma d\sigma$: $dA + A \wedge A = 0$

Integral of motion: $U = \mathcal{P}e^{\int d\sigma A_\sigma}$ $\frac{d}{d\tau}U = 0$ suitable b.c.

Infinitely many A-s $\leftrightarrow$ Infinitely many U-s $\rightarrow$ generating function $U(\lambda)$

Construct $A(\lambda)$ from the symmetry currents of the theory

$AdS_5 \times S^5$-coset space but action is not a 2d coset model

Noether currents: P-coset currents Q, Q'-supersymmetry

$$\downarrow$$

$$p = GPG^{-1} \quad q = GQG^{-1} \quad q' = GQ'G^{-1}$$

$$A(\lambda) = \alpha(\lambda)p + \beta(\lambda) * p + \gamma(\lambda)q + \delta(\lambda)q'$$

Expand $U(\lambda)$: – leading order: $AdS_5 \times S^5$ superisometries J^A

– nonlocal: $Q_2 = \int dx \int_x dy J_\sigma(x) J_\sigma(y) - 2 \int dx J_\tau(x)$

AdS/CFT: Q_2 (and higher) should have a gauge theory analog

Expand $U(\lambda)$: – leading order: $AdS_5 \times S^5$ superisometries J^A

– nonlocal: $Q_2 = \int dx \int_x dy J_\sigma(x) J_\sigma(y) - 2 \int dx J_\tau(x)$

AdS/CFT: Q_2 (and higher) should have a gauge theory analog

Test: SYM exhibits a different kind of integrable structure: the action of the 1-loop dilation operator on gauge invariant operators can be described in terms of an integrable spin chain (Minahan, Zarembo, Beisert, Staudacher)

– Construct the Yangian algebra (Dolan, Nappi, Witten; Belavin; Drinfeld)

$$Q_2^A = f_{BC}{}^A \sum_{i<j} J_i^B J_j^C \qquad\qquad [J^A, Q_2^B] = f^{AB}{}_C Q_2^C$$

– test compatibility with the spin chain Hamiltonian

$$\begin{cases} \delta\left([Q_2,\, \Delta] \sim Q_2\right) \\ [\delta Q_2,\, \Delta] \sim \delta Q_2 \end{cases} \quad \rightarrow \quad [Q_2, \delta\Delta] = 0$$

• AdS/CFT: string excitations $\longleftrightarrow$ gauge invariant operators

• Open string theory: SYM + infinitely many massive fields

Question: Is there a string theory whose spectrum and interactions are those of a Yang Mills theory?

Witten: Yes.

-Nair: MHV amplitudes: correlation function of fermionic currents

-Witten: large number of examples: tree level and some 1-loop amplitudes are supported on curves when transformed to the twistor space of Minkowski space

$p_{\alpha\dot{\alpha}} = p_\mu(\sigma^\mu)_{\alpha\dot{\alpha}}$ $\quad p^2 = \det p = 0 \;\; \Rightarrow \;\; p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} \;\rightarrow$ Fourier transform

e.g. MHV

$$\delta^4(\sum_i \tilde{\lambda}_i \lambda_i) f(\lambda) \quad \rightarrow \quad \int d^4x \prod_i \delta(\mu_i - x \cdot \lambda_i)\, f(\lambda)$$

Conjectured s.t.: $\begin{cases} \text{for } \mathcal{N}=4 \text{ SYM : lives on } CP^{3|4} \\ \text{open} \\ \text{topological} \rightarrow \text{finite spectrum} \\ \text{complete : perturbative} + \text{nonperturbative} \end{cases}$

perturbative: self-dual $\mathcal{N}=4$ SYM; lives on D5 brane in $CP^{3|4}$

− same spectrum as $\mathcal{N}=4$ SYM, $\mathcal{A}$, but different interactions

nonperturbative: space-time "instantons"

- D1 brane; free fields on their 2d world volume; couple as $J \wedge \mathcal{A}$

- can be morally identified with Nair's fermions

recover $\mathcal{N}=4$ scattering amplitudes by "integrating out" the 2d fields in $\mathcal{A}$ background

What amplitude? $\leftrightarrow$ property of D1 brane:

degree $\leftrightarrow$ $-$ helicity gluons

genus $\leftrightarrow$ number of loops

$$A_n = \int d\mathcal{M}_{g,d} \prod_{i=1}^{n} d\sigma_i \, \langle J(\sigma_1) \ldots J(\sigma_n) \rangle_{C_{d,g}}$$